LIQUID-DROP TYPE EXPANSION OF NUCLEAR BINDING AND DEFORMATION ENERGIES WITH SKYRME FORCES [+)]

M. Brack, C. Guet [++)], H.-B. Håkansson

Institute for Theoretical Physics, University of Regensburg, W.-Germany

and A. Magner, V.M. Strutinsky

Institute for Nuclear Research, 252 028 Kiev, U.S.S.R.

Abstract

Starting from the energy density formalism with effective Skyrme forces, we show how a liquid drop model like expansion of the nuclear binding energy can be systematically obtained. For a model Skyrme force with constant effective nucleon mass, all liquid drop parameters of symmetric nuclei can be given analytically in terms of the Skyrme parameters. Numerical tests of the leptodermous expansion are presented.

1. Introduction

The recent development of Hartree-Fock (HF) calculations with effective nucleon-nucleon forces of the Skyrme type [1)] on one hand and of the extended Thomas-Fermi (ETF) model for nonlocal potentials with variable effective mass and spin-orbit terms [2,3)] on the other hand have made it possible to describe average nuclear bulk properties like binding and deformation energies, radii and density distributions purely semiclassically using the energy density formalism. As was recently shown [4)], average HF results of such quantities are hereby reproduced rather accurately without the use of any adjustable parameters, but in a much more economical way since no wavefunctions have to be calculated. This allows us to make contact between the HF-Skyrme approach and liquid drop model parametrizations of average nuclear binding energies. More concretely, it is possible to determine the liquid drop parameters of a given Skyrme force rather accurately using the results of the semiclassical model.

In this paper we show how the use of an adequate system of curvilinear coordinates allows to obtain systematically the "leptodermous" expansion of the average binding energy, which holds as long as the surface diffusivity is small compared to the (local) curvature radius of the nuclear surface. From this expansion it is possible to discuss in detail the dependence of the binding energy both of nucleon number and of deformation.

Similar studies were already done some time ago [5,6)]. However, no sufficiently realistic energy density functional was available at that time. With the development of the Skyrme forces [1)] and the level of sophistication reached for the ETF density functionals [3,4)] to date, it seems worth while to resume this investigation once again, keeping in mind that one is still in need of more reliable sets of liquid drop parameters for the extrapolation to unknown regions of nuclei.

2. Intrinsic nuclear shape coordinate system

The experimental fact that the nuclear surface diffusivity is nearly independent of the nucleon number and thus of the curvature allows to approximate the nucleon densities to be functions essentially only of a coordinate along the normal direction to the equivalent sharp liquid drop surface, once its deformation has been fixed. It is thus natural to introduce a system of curvilinear coordinates such that two of the coordinates describe the sharp surface for each given shape of the nucleus, while the third measures the distance from the surface. Such a coordinate system has been introduced in ref. [5)]; for completeness we recall here its main features.

Restricting ourselves to axially symmetric nuclear shapes, we define $\rho = f(z')$ to be the generating curve for the sharp surface. The normal to the surface is the new variable u (positive outside and negative inside), and the variable z is the z' coordinate of the surface point ($u = 0$) (see Fig. 1).

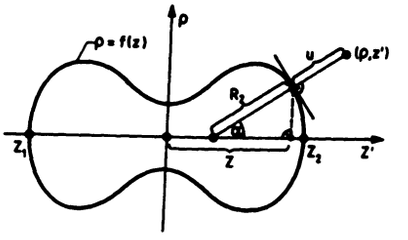

Fig. 1: Axially deformed nuclear shape.

Thus the ranges of these coordinates are $u \in [-R_2, +\infty)$ and $z \in [z_1, z_2]$. The volume element in the orthogonal set of coordinates (u, z, φ) is

$$dV = g(u,z)\,du\,dz\,d\varphi \qquad (1)$$

with the Jacobian

$$g(u,z) = \left(1 + \frac{u}{R_1}\right)(u + R_2)\,; \qquad (2)$$

R_1 and R_2 are the main curvature radii.

Our approximation to the semiclassical nucleon densities is that they only depend on the variable u. As found in the variational cacluations of ref. [4)], Fermi functions minimize rather well the semiclassical Skyrme energy in the spherical case. We thus parametrize the densities as follows

$$\left.\begin{aligned} \rho_n(\vec{r}) &= \rho_n(u) = \frac{\rho_{no}}{1+\exp\left(\frac{u+\Delta}{a_n}\right)}\,, \\ \rho_p(\vec{r}) &= \rho_p(u) = \frac{\rho_{po}}{1+\exp\left(\frac{u-\Delta}{a_p}\right)}\,. \end{aligned}\right\} \qquad (3)$$

Thus, we allow the protons and neutrons to have different (but "parallel") surfaces with a radius difference 2Δ. This leaves us, together with an overall

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++) Permanent address: D.R.F., D.P.N., C.E.N. Grenoble, France

scale parameter (e.g. $2c = z_1 - z_2$), with 6 parameters, two of which are eliminated by the particle number conservation. The other 4 parameters are determined variationally by minimizing the total energy (see eq. 5 below). In calculating the Skyrme energy density $\varepsilon[\rho_n,\rho_p]$ with the semiclassical kinetic energy density functionals $\tau[\rho]$ and $\vec{J}[\rho]$ (see refs. [2,3]), we need also the first and second derivatives of $\rho_q(\vec{r})$. In the present coordinates, one obtains simply

$$|\nabla\varrho|^2 = [\varrho'(u)]^2, \qquad \Delta\varrho(u) = \varrho''(u) + \left[\frac{1}{u+R_1} + \frac{1}{u+R_2}\right]\varrho'(u). \tag{4}$$

3. Liquid drop expansion of the energy for symmetrical nuclei

We now proceed to calculate the total energy for symmetric nuclei (with $\rho_n = \rho_p = 1/2\rho$):

$$E = 2\pi \int_{z_1}^{z_2} dz \int_{-R_2}^{\infty} g(u,z)\,\varepsilon[\varrho(u)]\,du \tag{5}$$

in the so-called "leptodermous" approximation, i.e. in the limit where the radius R_2 is everywhere large compared to the surface diffusivity $\alpha = \alpha_p = \alpha_n$:

$$R_2 \gg \alpha. \tag{6}$$

Then the lower limit of the u-integration in eq. (5) (which, in fact, is the only reason for the integral not to separate in u and z!) can be practically replaced by $-\infty$ for all contributions except the one which gives the volume energy. We therefore separate the latter out in writing

$$E = a_v A + \Delta E, \quad a_v = \varepsilon[\varrho_0]/\varrho_0, \tag{7}$$

where ρ_0 is the density at the centre. The correction ΔE is then approximately

$$\Delta E \approx 2\pi \int_{z_1}^{z_2} dz \int_{-\infty}^{\infty} du\, g(u,z)\left\{\varepsilon[\varrho] - a_v\varrho(u)\right\}. \tag{8}$$

Since the integral now can be separated, we obtain

$$\Delta E = b_0 S + 2b_1 C + 4\pi b_2, \tag{9}$$

where S and C are the sharp drop surface area and total mean curvature, respectively.

The coefficients b_0, b_1 and b_2 can be obtained once for all for a given energy density as functions of ρ_0, α and the force parameters and do not depend on deformation and nucleon number A. The main A dependence (except a very weak one through ρ_0 and α) and the entire deformation dependence are contained in S and C.

The explicit expressions for the coefficients b_n are given in the following. Because of the explicit z dependence (through R_1, R_2) of the Laplacian $\Delta g(u)$ eq. (4) we have to split up the total energy density. Writing

$$\varepsilon[\varrho] = \mathcal{F}(u) + \mathcal{G}(u)\Delta\varrho + \mathcal{H}(u)[\Delta\varrho]^2, \tag{10}$$

where the functions, $\mathcal{F}$, $\mathcal{H}$ and $\mathcal{G}$ may only depend on ρ and $\nabla\rho$, one gets

$$\begin{aligned} b_0 &= \int_{-\infty}^{+\infty}\left\{\mathcal{F}(u) + \mathcal{G}(u)\varrho''(u) + \mathcal{H}(u)[\varrho''(u)]^2 - a_v\varrho(u)\right\}du; \\ b_1 &= \int_{-\infty}^{+\infty}\left\{u\mathcal{F}(u) + \mathcal{G}(u)\varrho'(u) + 2\mathcal{H}(u)\varrho'(u)\varrho''(u) - ua_v\varrho(u)\right\}du; \\ b_2 &= \int_{-\infty}^{+\infty}\left\{u^2\mathcal{F}(u) + 2u\varrho'(u)[\mathcal{G}(u) + 2\mathcal{H}(u)\varrho''(u)] + 4\mathcal{H}(u)[\varrho'(u)]^2 - u^2a_v\varrho(u)\right\}du. \end{aligned} \tag{11}$$

In eqs. (11), the primes on ρ denote derivation with respect to u. In arriving at this result, we have neglected a term

$$2\pi\int_{z_1}^{z_2} dz\,\frac{1}{R_1}\int_{-R_2}^{\infty} du\,\mathcal{H}(u)\,[\varrho'(u)]^2\left(\frac{R_1-R_2}{u+R_2} + \frac{R_2-R_1}{u+R_1}\right) \tag{12}$$

whose deformation dependence is more complicated than that of the above simple invariants C,S or a constant. However, this term is seen to be exactly zero in the spherical case and negligibly small otherwise.

In order to sort out the deformation and nucleon number (A) dependence of S and C, we must now impose the conservation of A. We introduce in the usual manner the shape functions B_S and B_C, which are normalized to unity for the spherical case, by defining

$$S = 4\pi R_{1/2}^2\, B_S, \quad C = 4\pi R_{1/2}\, B_C. \tag{13}$$

With that we get

$$A \approx \frac{4\pi}{3}\varrho_0\left[R_{1/2}^3 + \pi^2\alpha^2 R_{1/2} B_C\right], \tag{14}$$

from where we find in a very good approximation

$$R_{1/2} \approx r_0 A^{1/3} - \frac{\pi^2\alpha^2}{3r_0}A^{-1/3}B_C. \tag{15}$$

Here the radius r_0 is defined by $r_0 = (3/4\pi\rho_0)^{1/3}$. We now can insert $R_{1/2}$ eq. (15) into eqs. (14) and arrive with eqs. (7,9) at the following expansion of the total energy:

$$E = a_v A + a_s A^{2/3} + a_c A^{1/3} + a_c + O(A^{-1/3}), \tag{16}$$

where

$$a_s = 4\pi r_0^2 B_S b_0, \tag{17}$$

$$a_c = 8\pi r_0 B_C b_1, \tag{18}$$

$$a_0 = 4\pi b_2 - \frac{8}{3}\pi^3\alpha^2 B_S B_C b_0. \tag{19}$$

The terms of order $A^{-1/3}$ and lower contribute less than 1 MeV to the total energy of heavy nuclei and are thus practically unimportant. The deformation dependence of the energy (16) is entirely contained in the shape functions B_S and B_C. The main A dependence of the energy is also separated out in eq. (16); a very smooth variation is, however, still coming from the b_n and r_0 through their dependence on ρ_0 and α.

4. Dependence of the central density ρ_0 on the nucleon number A

The derivation of the central density ρ_0 of finite nuclei from the saturation density ρ_∞ of infinite nuclear matter has traditionally been estimated from the saturation condition itself [5,6]. The latter is automatically fulfilled if we write the energy density of infinite nuclear matter in the form

$$\varepsilon_\infty(\rho) = \rho\left[a_v^\infty + \frac{1}{18}K_\infty\left(\frac{\rho-\rho_\infty}{\rho_\infty}\right)^2\right]. \quad (20)$$

Here K_∞ is the nuclear incompressibility. For a given Skyrme force a_v^∞, ρ_∞ and K_∞ are uniquely given constants. Now, the variational equation for a finite symmetric nucleus (without Coulomb force), viz.

$$\delta\int d^3r\left\{\mathcal{E}[\rho] - \lambda\rho(\vec{r})\right\} = 0, \quad (21)$$

with the Lagrange multiplier λ for nucleon number conservation, can be solved easily at the centre of the nucleus using the functional $\varepsilon_\infty(\rho)$ eq. (20), if the surface contributions are all exponentially small there. Then, the only parameter to be varied is ρ_0, and eq. (21) takes the form

$$\left.\begin{aligned} &a_v^\infty - \lambda - \frac{K_\infty}{3}\epsilon + \frac{3}{2}K_\infty\epsilon^2 = 0; \\ &\epsilon = -\frac{1}{3}\,\frac{(\rho_0-\rho_\infty)}{\rho_\infty}. \end{aligned}\right\} \quad (22)$$

The A dependence thus comes in only through the Fermi energy λ.

Expanding the coefficients a_V and a_S around the saturation density ρ_∞ one gets up to order $A^{-2/3}$

$$\epsilon = \frac{1}{9} - \frac{1}{9}\sqrt{1 + \frac{36}{K_\infty}\left(a_S^\infty A^{-1/3} + \frac{1}{2}a_C^* A^{-2/3}\right)} \quad (23)$$

with

$$a_C^* = a_C^\infty + \frac{2}{K_\infty}\,a_S^\infty\left[a_S^\infty + 3\rho_\infty a_S'(\rho_\infty)\right]. \quad (24)$$

As we will show below eq. (23) does not reproduce very well the central density ρ_0 found numerically with the energy density method. The reason for this is not the inadequacy of the leptodermous expansion as such, but the fact that the Fermi type densities used above (and in the droplet model) are not exactly solutions of the Euler equations (21). Thus, the <u>local</u> variation at the centre, eq. (22), does not lead to the same result as the <u>global</u> variation of the total energy with respect to the parameters of the Fermi function.

It is therefore more consistent to derive the central density ρ_0 from the variation of the <u>total</u> energy as obtained in the leptodermous expansion in eq. (16). Neglecting the α dependence (which is very weak, as shown below), we thus write

$$\frac{dE}{d\rho_0} = A\frac{da_V}{d\rho_0} + A^{2/3}\frac{da_S}{d\rho_0} + \ldots = 0. \quad (25)$$

Expanding a_V, a_S and a_C as above up to terms of order $A^{-2/3}$, one obtains

$$\epsilon = \frac{\rho_\infty\left[a_S'(\rho_\infty)A^{-1/3} + a_C'(\rho_\infty)A^{-2/3}\right]}{3\left\{\frac{K_\infty}{9} + \rho_\infty^2\left[a_S''(\rho_\infty)A^{-1/3} + a_C''(\rho_\infty)A^{-2/3}\right]\right\}}. \quad (26)$$

With eq. (26), the central densities found numerically are reproduced within less than $1.0^{-3}\rho_0$ for $A \gtrsim 100$, thus demonstrating the validity of the leptodermous expansion.

Based on these results, it is easy to derive an expression for the incompressibility of finite nuclei, which we here define as

$$K_A = 9\rho_0^2\frac{d^2}{d\rho_0^2}\left(\frac{E}{A}\right) \quad (27)$$

(see e.g. ref. [7]). In expanding K_A consistently up to order $A^{-1/3}$ one arrives at

$$K_A = K_\infty + 9\left[\rho_\infty^2 a_S'' - 2\rho_\infty a_S' - \frac{9}{K_\infty}\rho_\infty^4 a_S' a_V'''\right]A^{-1/3} + O(A^{-2/3}). \quad (28)$$

We will see in the next section that this expression agrees very well with the numerical results for K_A eq. (27).

5. Numerical tests for symmetric nuclei

Before presenting some quantitative tests of the quality of the leptodermous approximation, we shall write down some explicit expressions using the traditional parametrizing of Skyrme forces, restricting ourselves hereby to symmetric nuclei without Coulomb interaction. The energy density then has the explicit form [1]

$$\mathcal{E}[\rho] = \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^3 + \frac{1}{64}(9t_1 - 5t_2)(\nabla\rho)^2 + \frac{1}{16}(3t_1+5t_2)\tau\rho + \frac{3}{4}W_0\,\vec{\nabla}\rho\cdot\vec{J}. \quad (29)$$

The semiclassical functionals $\tau[\rho]$ and $\vec{J}[\rho]$ developed to 4th order in the $\hbar$ parameter are given in ref. [3]. The integrals entering the definition of the surface tension b_0 eq. (11) can all be done analytically [3]. This is no longer so for some contributions to the parameters b_1 and b_2 in eq. (11). However, for a force with constant effective nucleon mass m* = m (i.e. if $\beta = 0$), all the leptodermous integrals in eq. (11) can be done once for all and their explicit dependence on the density parameters ρ_0 and α is known. Such a force has been published with the label SVII [8] and used in the present numerical calculations.

Fig. 2 shows the values of ρ_0 and α obtained for spherical nuclei with the Skyrme force SVII as functions of the mass number A. The variation of α, less than 1 % over the range considered here, is so weak for the present case that we may replace it by

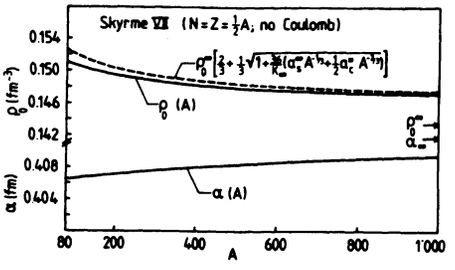

Fig. 2: Central density ρ_0 and surface diffuseness α versus nucleon number A for symmetrical nuclei.

the value $\alpha_\infty = 0.412$ (obtained by minimizing the surface tension b_0 at $\rho_0 = \rho_\infty$, which easily is done analytically), without affecting the total energies by more than $\sim$ 0.5 MeV. Then, the determination of ρ_0 can be done as described in sect. 4 above; the result

quoted in eq. (26) then still reproduces the exact numerical values $\rho_0(A)$ within $\sim 1\ ^0/_{00}$ for A > 100. The dashed line in Fig. 2 shows the central density obtained by applying the variational equation locally at the centre of the nucleus, leading to the usual droplet model value [6] $\varepsilon = -2a_S^\infty A^{-1/3}/K_\infty$.

The values of the integrals eq. (11), taken at $\rho_0 = \rho_\infty$, are

$$b_0 = 0.9972\ \text{MeV fm}^{-2},$$
$$b_1 = 0.3347\ \text{MeV fm}^{-1}, \qquad (30)$$
$$b_2 = 0.6692\ \text{MeV}.$$

From them and their variations with ρ_0 we find (all quantities in MeV):

(31)

$$a_S^\infty = 17.61,\quad \rho_\infty a_S'(\rho_\infty) = -10.8,\quad \rho_\infty^2 a_S''(\rho_\infty) = -84.6;$$

$$a_c^\infty = 9.97,\quad \rho_\infty a_c'(\rho_\infty) = 22.0,\quad \rho_\infty^2 a_c''(\rho_\infty) = 39.4;$$

$$a_0^\infty = -5.65,\quad \rho_\infty a_0'(\rho_\infty) = -1.1,\quad \rho_\infty^2 a_0''(\rho_\infty) = 39.2.$$

With that, the leptodermous expansion of the total energy eq. (16), after expanding out the A dependence using eq. (26), becomes

$$E = a_v^\infty A + a_S^\infty A^{2/3} + a_c^* A^{1/3} + a_0^*. \qquad (32)$$

The expression for a_c^* has already been given in eq. (24). The quantity a_0^* receives many contributions from expanding a_V, a_S and a_C around ρ_∞; its expression in terms of the quantities in eq. (31) is straightforward but cumbersome and has not much practical value, since the parameter a_0^* can not be determined reasonably well in any least-squares fit, as we shall see below.

In Table 1 we list the 4 parameters of eq. (32) obtained in the leptodermous approximation along with the results of several least-squares fits to the total exact energies (obtained numerically from the variational calculation without any further approximation). We see that, apart from the ambiguities of such fits themselves (especially concerning the constant term a_0^*!), there is an excellent agreement. This shows, that one is in principle able to determine sufficiently accurately the three leading terms of the expansion (32) for a given Skyrme force just in terms of the simple 1-dimensional integrals eq. (11).

To test the validity of the leptodermous expansion as a function of the deformation, we have made a calculation for A = 240 using the shape parametrization (c,h) used in connection with fission barrier calculations [9]. In Table 2 we present various quantities as a function of the elongation parameter c(h=0). The spherical shape corresponds to c = 1, the liquid drop saddle point of actinide nuclei to $c \simeq 1.5$. The second column shows the neck radius of the corresponding shape which is strongly constricted at c=2.0. The next three columns contain the shape functions B_S and B_C and the radius scaling parameter $R_{1/2}$, from which the surface area S and the total mean curvature C are obtained via eqs. (13). It is worth noting that, with the values $\rho_0 = 0.1492\ \text{fm}^{-3}$, $\alpha = 0.408$ fm valid for A = 240, the leptodermous result $R_{1/2}^{lept}$ eq. (15), shown in the next column, reproduces the exact values within $\sim 10^{-4} R_{1/2}$ even at the largest deformation. In columns 7 and 8 we show the total energy minus the volume energy and the leptodermous result eq. (9) for ΔE, respectively; their difference is shown in the last column. Apart from a constant error of ~ 10 MeV, which reflects the difficulties in determining the constant term a_0^* of the energy expansion eq. (32) observed above, there is only a small variation of $\lesssim 1.3$ MeV over the whole range of deformation. This is rather astonishing, observing the small neck radius of 2.26 fm at the largest deformation. Up to typical saddle point deformations (c < 1.6), the error in the deformation energy brought about by the leptodermous expansion is even not larger than 0.3 MeV.

Table 1: Liquid drop parameters of the expansion eq. (32) of the total energy obtained in the exact variational calculation for symmetric nuclei with the force SVII (no Coulomb). (All results in MeV). The first line gives the parameters obtained in the leptodermous expansion. The others give the results of different least-squares fits to the exact results for 24 nuclei ranging from A = 80 to A = 1000 (except in the last line for only 21 nuclei with $200 \lesssim A \lesssim 1000$). σ is the root mean square deviation in MeV. a_{-1} and a_{-2} are the coefficients of terms proportional to $A^{-1/3}$ and $A^{-2/3}$, respectively. a) Value fixed in the fitting to 24 nuclei. b) Value fixed in the fitting to 21 nuclei.

σ	a_v^∞	a_s^∞	a_c^*	a_0^*	a_{-1}	a_{-2}
leptod.	- 15.782	17.61	8.53	- 2.74	-	-
0.004	- 15.789	17.56	8.57	- 10.01	4.6	8.7
0.06	- 15.778	17.34	9.81	- 10.81	0[a]	0[a]
3.45	- 15.782[a]	17.61[a]	8.53[a]	- 16.24	0[a]	0[a]
0.51	- 15.782[a]	17.61[a]	6.40	0.47	0[a]	0[a]
0.30	- 15.782[b]	17.61[b]	6.17	2.48	0[b]	0[b]

Table 2: Various quantities obtained for A = 240 with force Skyrme SVII versus elongation c (see text for details).

c	neck radius (fm)	B_s	B_c	$R_{1/2(fm)}$	$R^{lept.}_{1/2}$ (fm)	$E - a_v A$ (MeV)	$\Delta E_{lept.}$ (MeV)	error (MeV)
1.0	7.19	1.0	1.0	7.1935	7.1939	712.3	722.1	9.8
1.2	6.45	1.0151	1.0168	7.1922	7.1927	723.0	733.0	10.0
1.4	5.73	1.0546	1.0654	7.1885	7.1890	751.7	761.7	10.0
1.6	4.93	1.1146	1.1486	7.1822	7.1828	796.0	806.1	10.1
1.8	3.90	1.1925	1.2750	7.1725	7.1733	854.5	865.0	10.5
2.0	2.26	1.2745	1.4730	7.1576	7.1586	920.0	931.1	11.1

In order to test finally the expression given above for the incompressibility of finite nuclei K_A, eq. (28), we present in Fig. 3 the numerical results of K_A according to eq. (27) with the SVII force. The behaviour of K_A versus $A^{-1/3}$ is linear:

$$K_A = K_\infty - A^{-1/3} \times 565 \text{ MeV}. \qquad (33)$$

With the values in eq. (31) and with $\rho_\infty^3 a_V''' = 5.8$ MeV for SVII we obtain for the leptodermous expansion eq. (28)

$$K_A = K_\infty - A^{-1/3} \times 553 \text{ MeV} - A^{-2/3} \times 6 \text{ MeV} \qquad (34)$$

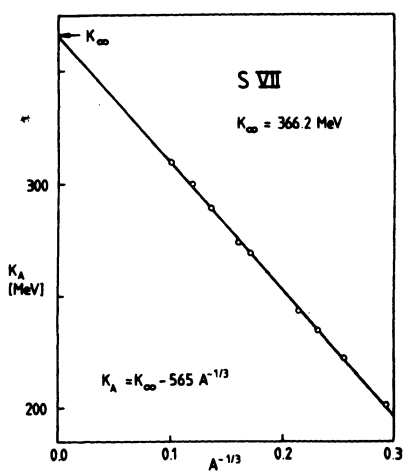

Fig. 3: Incompressibility K_A for symmetric nuclei versus particle number A.

6. Realistic Skyrme forces and droplet model parameters

For realistic nuclei with N≠Z and the Coulomb interaction included, the above expansion becomes much more elaborate, although in principle it could be carried through. Rather than doing that, we shall take the above results for the symmteric case to be a strong evidence for the fact that the Skyrme force plus energy density approach is completely equivalent to an extended liquid drop type approach (including curvature terms).

Thus, we shall use the energies obtained numerically with the semiclassical model [4,10] for realistic cases and fit them by an energy functional of the form

$$E(A,I) = a_v(1-\kappa_v I^2)A + a_s(1-\kappa_s I^2)A^{2/3} + a_c(1-\kappa_c I^2)A^{1/3} + a_0 + E_{coul.}, \qquad (34)$$

which suggests itself from the result [4,10] that the energies of asymmetric nuclei are proportional to $I^2=((N-Z)/A)^2$ even up to values of I = 0.4 - 0.5. The volume energy, a_v, and the volume symmetry energy, $a_v\kappa_v$ are uniquely given for each force and need thus not to be fitted. In terms of droplet model parameters we write

$$-a_v\kappa_v = +J, \quad a_s\kappa_s = \frac{9}{4}\frac{J^2}{Q}, \qquad (35)$$

which means that we do not use the full expression for the surface asymmetry in the droplet model. Instead we make the usual expansion of the central density asymmetry

$$\delta = \frac{I}{1+\frac{9}{4}\frac{J}{Q}A^{-1/3}} \to I\left[1-\frac{9}{4}\frac{J}{Q}A^{-1/3}\right] \qquad (36)$$

which, as will be seen below, is justified for the values of Q here encountered.

An important point in most droplet model studies is that the curvature corrections are neglected. This is at variance with our model using Skyrme forces, as visible in fig. 4 where we have plotted the quantity $(E/A-a_v)A^{1/3}$ against $A^{-1/3}$. The ordinate at origion provides the surface energy, a_s, while the slope measures the curvature energy, a_c, which clearly here is <u>not</u> zero.

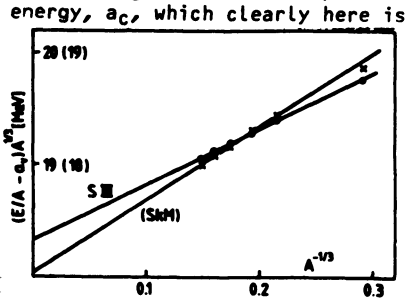

Fig. 4: Determination of the surface and curvature energies for the forces SIII and SkM. The SkM force was introduced in ref. [11].

The surface asymmetry coefficient is determined by using the fact that the central density asymmetry $\delta = (\rho_n^0 - \rho_p^0)/(\rho_n^0 + \rho_p^0)$ depends linearly on the isospin I when the total nucleon number, A, is kept constant, see eq. (36). This linear behaviour predicted by the droplet model is also found to be exact in our energy density formalism, see fig. 5.

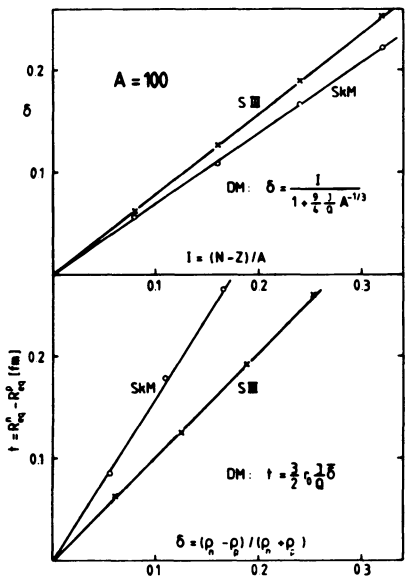

Fig. 5: Two ways of determining the surface asymmetry coefficient Q.

(For the sake of clarity the Coulomb interaction is not included.) An alternative way of getting the surface asymmetry coefficient Q is to look at the dependence of the neutron skin thickness $t(t = R_n - R_p$, where the radii are the equivalent sharp surface radii) upon the variable δ, which according to the droplet model is also linear:

$$t = \frac{3}{2} r_0 \frac{I}{Q} \delta ; \qquad (37)$$

a statement here confirmed. The value of Q extracted from the slope $dt/d\delta$ is in perfect agreement with the previous estimation (see fig. 5). Finally the curvature asymmetry coefficient can also be, with a larger uncertainty however, deduced from the slope dE/dI^2 for a given A.

In table 3 we have written all the coefficients we calculated by these simple methods and compared them to the output of a least squares fit of the function $E(A,I^2)$ eq. (34) to the calculated binding energies of a randomly chosen ensemble of nuclei. Note that the root-mean-square-errors (RMS) for these fits are less than 0.06 MeV. A good agreement between the two methods is observed, thus unambigously showing the strong analogy between the elaborated droplet model and the energy density formalism such as the present ETF model.

7. Conclusions and outlook

Using the leptodermous expansion, we have demonstrated that the energy density formalism using Skyrme forces allows a quantitative determination of the droplet model type parameters for symmetric nuclei. For asymmetric nuclei including the Coulomb interaction, the analysis becomes more cumbersome, since different density parameters $R_{1/2}$, ρ_0, α have to be used for protons and neutrons. Recent results have shown that the difference in the diffuseness parameters $\alpha_n - \alpha_p$ is clearly correlated to the isospin (N-Z)/A of the nucleus [4], which is an effect not included in the droplet model [6]. Apart from that, the essential droplet model relations for the asymmetry parameters, in particular the effective surface stiffness Q, seem to be fulfilled at least qualitatively. A detailed analysis of the leptodermous expansion in the asymmetric case is presently under way [10].

Concerning the deformation dependence of the total energy obtained in the semiclassical energy density method, the leptodermous expansion has been shown here to be very accurate even beyond typical saddle point deformations of heavy nuclei. A numerical test of the corresponding expansion of the Coulomb energy is actually under way; if it holds equally well we may conclude that a full variational calculation is not needed in the deformed case. Instead, it would be sufficient to calculate the droplet model parameters for a given force on one hand and the shape functions B_s, B_c and B_{coul} on the other hand once for all and then use eqs.(7),(9) (including the Coulomb energy) to obtain the deformation energy. Work along these lines is now in progress.

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References

1. D. Vautherin, D. Brink, Phys. Rev. C5(1972)626
2. M. Brack, B.K. Jennings, Y.M. Chu, Phys. Lett. 65 B (1976) 1
3. B. Grammaticos, A. Voros, Ann. of Phys. 123 (1979) 359; 129 (1980) 153
4. C. Guet, H.-B. Håkansson, M. Brack, Phys. Lett. 97 B (1980) 7
5. V.M. Strutinsky, A.S. Tyapin, Sovj. Phys. JETP 18 (1964) 664
6. W.D. Myers, W.J. Swiatecki, Ann. of Phys. 55 (1970) 395; 84 (1974) 186
7. W. Stocker, Nucl. Phys. A 342 (1980) 293
8. M.J. Giannoni, P. Quentin, Phys.Rev.C21(1980)2076
9. M. Brack et al., Rev. Mod. Phys. 44 (1972) 320
10. C. Guet, H.-B. Håkansson and M. Brack, to be published
11. H. Krivine, J. Treiner, O. Bohigas, Nucl. Phys. A 366 (1980) 155

Table 3: Droplet model parameters of the forces SIII and SkM. The stars indicate that these quantities have been calculated for infinite nuclear matter. The four last columns have been obtained by a least square fit of the function eq. 34 (upper lines), or by the graphical procedures described in the text and figs. 4, 5 (lower lines). ($a_{cs} = -a_c \kappa_c$)

Force	k_F^*[fm^{-1}]	a_v^*[MeV]	J^*[MeV]	a_s[MeV]	Q[MeV]	a_c[MeV]	a_{cs}[MeV]
SIII	1.29	-15.86	28.16	18.30 18.3	49.4 50	4.87 5.0	23.4 21.3
SkM	1.33	-15.78	30.75	17.18 17.2	38.9 35.5	6.09 6.4	23.7 36.5